Method of Optimizing the Update Intervals in Hybrid Navigation Systems

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Introduction

THE effectiveness of hybrid navigation systems[1] can be increased by properly selecting the update intervals between the individual systems of the complete system. The following method shows that two bounds on the update intervals can be specified numerically. The two bounds depend on the user's peformance requirements. The final decision is a tradeoff between the two previous bounds and computer capability.

Two Bounds on the Update Intervals

Hybrid navigation systems are complex nonlinear systems.[2,3] The noise disturbances are assumed to be white and enter the system in a linear fashion. We start with the following model:

$$\dot{x}(t) = f[x(t),t] + G(t)W(t) \tag{1}$$

$$z(k) = h[x(k),k] + V(k) \tag{2}$$

where $x(t)$ is the state vector, $z(k)$ is the measurement vector received at time t_k, $W(t)$ is white Gaussian noise with zero mean and covariance $Q(t)\delta(t-\tau)$, $V(k)$ is white Gaussian sequence with zero mean and covariance $R(k)\delta_{jk}$, and f and h are nonlinear functions. In addition, $\delta(\cdot)$ is the Dirac delta and δ_{jk} the Kronecker delta.

After linearization and discretization, we find

$$\delta x(k+1) = \Phi(k+1,k)\delta x(k) + W(k) \tag{3}$$

$$\delta z(k) = H(k)\delta x(k) + V(k) \tag{4}$$

where δx and δz are variations about certain nominal trajectories $\bar{x}$ and $\bar{z}$ and $W(k)$ is a white Gaussian sequence.[4]

Now, $\hat{x}$ an estimate of x, $\tilde{x}$ the estimation error, and P the covariance of $\tilde{x}$, are defined. Measurements are received at time points $t_1, t_2, \ldots, t_k$. At time t_k $\hat{x}(k|k)$ and $\hat{x}(k+1|k)$ are computed (computation takes δ_1s) using Kalman filtering techniques.[4,5] At time t_{k+1}, the measurements $z(k+1)$ are available and $\hat{x}(k+1|k+1)$ is computed. Clearly, $\Delta t = t_{k+1} - t_k$ has to be greater than δ_1. Thus,

$$\Delta t > \delta_1 \tag{5}$$

By increasing Δt the accuracy of the model is reduced because higher than first-order terms cannot be ignored when it is linearized. Also, keeping the system with no information for a long time increases the prediction time and the prediction error covariance matrix $P(k+1|k)$. But

$$P(k+1|k+1) = [I - K(k+1)H(k+1)]P(k+1|k)$$

from which it can be seen that $P(k+1|k+1)$ also increases.

The trace of the matrix $P(k|k)$ is considered and required to be less than a certain bound C which is defined by the user's accuracy requirements. Then

$$\mathrm{tr}P(k|k) < C \tag{6}$$

The objective is to find a Δt which satisfies Eqs. (5) and (6). This can be done numerically by increasing or decreasing Δt. The obvious choice of Δt would be the smallest possible; however, a lower bound can be found.

In practice, the continuous noise process is not white or, equivalently, has finite bandwidth. Sampling at time points spaced by Δt is analogous to passing the continuous noise through a bandpass filter of bandwidth $B = 2\pi/\Delta t$. We know that the autocorrelation function of such bandpass noise process is

$$R(\tau) = A\sin \tfrac{1}{2}BT / \tfrac{1}{2}BT$$

where A is constant. $R(\tau)$ has its first zero at $\tau = \pi/\tfrac{1}{2}B$ or, for our case, at $\tau = \pi/(\pi/\Delta t) = \Delta t$. Consequently, by sampling at points spaced by Δt, uncorrelated Gaussian noise samples are obtained. Implicit in this discussion is the fact that the continuous nonwhite noise has a larger bandwidth than the filter bandwidth and, consequently, can herein be considered white. If the size of Δt is now reduced to improve accuracy, the bandwidth of our filter is increased because

$$\lim_{\Delta t \to 0} B = \lim_{\Delta t \to 0} (2\pi/\Delta t) = \infty$$

It is clear then that eventually a point is reached beyond which the noise bandwidth is smaller than the filter bandwidth and hence may no longer be considered white. This, in turn, means that the system model is not valid, and a lower bound for Δt is obtained. We now demonstrate how to find this lower bound numerically.

The system of Eqs. (3) and (4) is linear. The innovation process of this system

$$\delta\tilde{z} \triangleq \delta z - \delta\hat{z} = \delta z - H\delta\hat{x}$$

is a white Gaussian stochastic process. By definition,

$$\delta z(k) = z(k) - \bar{z}(k) = z(k) - h(\bar{x}(k),k) \tag{7}$$

We know $x(k)$ and we receive $z(k)$ at each point k, so $\delta z(k)$ can be generated using Eq. (7). From the equation

$$\hat{x}(k|k) = \bar{x}(k) + \delta\hat{x}(k|k)$$

$\delta\hat{x}(k|k)$ is specified and, consequently, $\delta\tilde{x}(k)$ is checked for whiteness. This is done by computing the correlation coefficients of the random numbers $\delta\tilde{z}(k)$. An estimate r_k of the

Table 1 Trace of the covariance matrix $p(k|k)$

	Time interval Δt, s				
Time, k	1	2	5	10	20
1	1.1121	1.6659	1.9365	1.9826	1.9946
2	1.2396	1.5686	1.8203	1.9130	1.9581
3	1.2255	1.5677	1.8032	1.9172	1.9595
4	1.2193	1.5679	1.8294	1.9170	1.9593

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Table 2 Residuals and correlation coefficients

Time, k	Residuals	Time, k	Residuals	Lag, Δt	r_k
1	0.50466	13	3.03277	Δt	0.24663
2	−0.88240	14	−1.24318	$2\Delta t$	0.05611
3	1.83704	15	−0.31517	$3\Delta t$	0.07838
4	−0.03073	16	−0.18640		
5	−0.18433	17	−0.53914		
6	1.06789	18	0.11952		
7	0.87981	19	−0.74797		
8	0.02979	20	0.04551		
9	1.58582	21	−0.83975		
10	−0.02825	22	−0.44318		
11	1.17027	23	0.24139		
12	−2.33596	24	−0.29270		

correlation coefficients at lag k is given by[6]

$$r_k = \frac{C_k}{C_0} \qquad C_k = \frac{1}{N}\sum_{t=1}^{N-k}(z_t - z_{av})(z_{t+k} - z_{av})$$

$$(k=0,1,2,\ldots)$$

and $k<N$, usually of the order of $N/4$, where N is the number of data points, z_t is the random number received at point t, z_{t+k} is the random number received at point $t+k\Delta t$, and z_{av} is the average of all z_t.

If $|r_k|<\gamma$, where γ is some positive constant depending on the accuracy requirements of the user, the Δt that gave us the above r_k is selected. If $|r_k|<\gamma$, then Δt is increased until $|r_k|<\gamma$ is satisfied.

Two simple examples illustrate the previous ideas. We consider the nonlinear system

$$\dot{x}_1 = \tfrac{1}{2}x_2^2 \tag{8}$$

$$\dot{x}_2 = w(t) \tag{9}$$

with $\bar{x}_2(0)=1$, $\bar{w}(k)=0$, and $E[w(i)w^T(k)]=0.2\delta_{jk}$. After linearization about $\bar{x}_2(t)=1$ and discretization, we find

$$\delta x(k+1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \delta x(k) + \begin{bmatrix} \tfrac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} w(k) \tag{10}$$

Let us assume that the measurement model is found as

$$\delta z(k+1) = \delta x_1(k) + v(k+1) \tag{11}$$

where $\bar{v}(k)=0$ and $E[v(j+1)v^T(k+1)]=\delta_{jk}$.

Finally, it is assumed that tr $P(kk)\leq 1.70$.

The results of these computations with $\Delta t = 1, 2, 5, 10, 20$ s are presented in Table 1. Clearly, $\Delta t=2$ s satisfies the user's requirements.

The second example concerns the numerical generation of Gaussian white noise with zero mean and variance one, according to one of the standard numberical methods. These numbers are assumed to be the residuals of the system of the previous example where we used $\Delta t=0.5$ s and $\gamma=0.06$. The results are shown in Table 2. We notice that for a shift $2\Delta t=1$ s, $|r_k|<0.06$ is obtained and, consequently, whiteness is quite good. Finally, if we take $1\leq\Delta t\leq 2$ s, the best performance of the system is achieved. If $\Delta t>2$, then the system's performance is below the user's requirements, although the model is valid. If $\Delta t<1$, the system's model is not valid. The importance of proper selection at Δt then becomes obvious.

Conclusion

The size of the discretization interval Δt effects the performance of hybrid navigation systems. Two bounds for the interval Δt can be specified numerically. Both bounds must be greater than the computation time, otherwise the best Δt is equal to the computation time.

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Concept to Enhance Strapdown Navigation on MaRV's

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Introduction

STRAPDOWN inertial measurement units (IMU's) have long been considered for maneuvering re-entry vehicle (MaRV) navigation because of their potentially lower life-cycle cost, greater reliability, and convenient source of body-frame rate and acceleration data. They are extremely difficult to calibrate, however, and because they must therefore rely on long-term stability of the inertial sensors for performance they have received considerably less attention than their gimballed counterparts for this class of application.

The accuracy is of particular concern for exospherically spin-stabilized MaRV's, where the major source of strapdown navigation error during re-entry is the roll angle error accumulated prior to re-entry. The dominant error source is the roll gyro's scale factor uncertainty. Several projects have been initiated or proposed by government and industry to reduce the uncertainty or mitigate its effect, for example: enhanced long-term stability of gyros (particularly laser gyros), addition of a roll-isolation gimbal, and incorporation of roll-update sensors such as star trackers, gravity gradiometers, magnetometers, horizon scanners, etc.

An alternate concept is presented which essentially requires only a procedural modification at the system level in eliminating the MaRV accuracy sensitivity to the roll angle error accumulated prior to re-entry. Following the boost phase, it is suggested that a MaRV containing a strapdown navigation system be oriented, deployed, and spin-stabilized along the effective or resultant re-entry "drag vector" direction rather than along the conventional re-entry velocity vector direction. For typical maneuvers, the "drag vector" is 10-20 deg away from the re-entry velocity vector. Since the CEP contribution due to roll error is a function of the cross product between the "drag vector" and the spin vector, then forcing them to be colinear results in (theoretically) zero position error.

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